Stochastic Moiré II

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Abstract

In color AM halftoning, moiré is the low frequency spatial artifact generated by the interference of superimposed primary color dot screens that adds an artificial texture to the printed image and renders the image visually unpleasing. When these overlapping dot screens that form FM halftones, this interference pattern follows a random spatial distribution resulting in "stochastic" moiré. In this paper, we show that stochastic moiré is at its most visible when two overlapping dither patterns have the same relative spacing between dots. We will also show the measured stochastic moiré visibility for several traditional error-diffusion algorithms.

1. Introduction

In inkjet printing where the variation in the printed dot is sufficiently small as to allow FM halftoning, the spatial distribution of dots is stochastic, as opposed to regular, and was thought to avoid the moiré phenomenon in color printing. But recently, it has been shown by Lau *et al*¹ that stochastic FM halftoning exhibits low frequency graininess that is created by fluctuations in texture for two overlapping FM patterns. These fluctuations in texture are referred to by Lau *et al* as *stochastic moiré*. For a demonstration of stochastic moiré, Fig. 1 shows two instances of two color (cyan and magenta) FM halftones.

In Fig. 1 (top), the halftoning algorithm creates large patches of "light" textures where cyan dots have a very high probability of overlapping a magenta dot and large patches of "dark" textures where cyan dots have a very low probability of overlapping magenta. In Fig. 1 (bottom), a different halftoning algorithm is used with the halftone showing much smaller patches of light and dark textures. Now in this figure (Fig. 1 (bottom)), the fluctuations in texture from light to dark occur at a much faster rate per unit length than in Fig. 1 (top), and Fig. 1 (bottom) is, therefore, composed of much higher spatial frequency content than Fig. 1 (top) because of this. The HVS, being less sensitive to high frequency spatial content, is less conscious of the fluctuations in texture occurring in Fig. 1 (bottom), and it is Fig. 1 (top) that is more likely to draw objections from the viewer. The stochastic moiré in Fig. 1 (top) is, therefore, considered to be more visible than in Fig. 1 (bottom).



Figure 1. Two color halftone patterns generated using error diffusion with the black and white diagrams indicating regions where the minority pixels of cyan and magenta do (black) and do not (white) overlap. These black and white diagrams were created by hand.

2. Spectral Analysis of Stochastic Moiré

Figure 2 shows several stochastic halftone patterns with each composed of 6.25% (1/16) cyan coverage and 6.25% magenta coverage. Shown in pattern **A** is the case where magenta pixels are placed directly on top of all cyan pixels. Pattern **C** shows the instance where minority pixels of cyan and magenta are homogeneously distributed such that magenta pixels are placed midway between cyan pixels. While it may be argued by a given observer that pattern **A** creates a hue different from that of pattern **C**, we ignore those differences focusing on the fact that pattern **A** certainly portrays a different texture from that of **C**. Any differences in hue are assumed to be corrected by manipulating the percentage of coverage of the various inks. Now while **A** and **C** have different textures, we make no claims as to whether A or **C** is a better texture.



Figure 2. Three two-color halftone patterns illustrating (A) perfectly overlapping or in-phase pixels, (B) uncorrelated pixels, and (C) perfectly non-overlapping or out-of-phase pixels.

Noting pattern **B**, the magenta and cyan dither patterns are completely uncorrelated with some minority pixels (of both patterns) overlapping and some falling directly in between those of the other. All other minority pixels fall some where in between these two extremes. From pattern **B**, we define stochastic moiré as the change in texture that occurs from one point to another within a given halftone. Like periodic moiré, the optimal stochastic halftone is the one that either minimizes or maximizes the amount of fluctuation in this texture per unit area. In the case of pattern **A** and **C**, the amount of fluctuation is minimized with patterns **A** and **C** offering equivalent optimality regarding moiré.

For a formal definition of stochastic moiré, Lau *et al*¹ define a discrete-space, 2-D function $\mathcal{D}[n]$ from the dither patterns $\phi_A = \{a_i : i = 1, 2, ..., N\}$ and $\phi_B = \{b_j : j = 1, 2, ..., M\}$ such that:

$$D[n] = \frac{1}{\lambda_b} \sum_{i=1}^{N} d_i \delta[n - a_i], \text{ where}$$
(1)

$$d_i = \min_i \left\| a_i - b_j \right\|_2 \tag{2}$$

In its eqn. (1) form, $\mathcal{D}[n]$ represents a continuous-space signal $\mathcal{D}(x)$, an image we will refer to as the *stochastic moiré surface*, that has been sampled on a stochastic sampling grid defined by ϕ_A .

If the sampling grid has a Poisson distribution, be the case of a blue-noise dither pattern with wavelength λ_a , we can obtain $\mathcal{D}(x)$ from $\mathcal{D}[n]$ ideal low-pass filter with cutoff frequency $0.5\lambda_a^{-1}$ a magnitude λ_a^2 , Fig. 3. The magnitude of

the low-pass derived from Papoulis² who shows that if x[n] is the sampled signal of x(t) defined by a Poisson sampling process with average density g, then the sum

$$\widetilde{X}(f) = \frac{1}{g} \sum_{n} x[n] e^{-j2\pi j n}$$
(3)

is an unbiased estimate of X(f), the Fourier transform of x(t), given that the aliases are nulled out. Hence Fourier transform of $\mathcal{D}[n]$ is scaled by 1/g or equivalently by λ_a^2 to obtain the unbiased estimate $\mathcal{D}(f)$.

Given our stochastic moiré surface, characterizing stochastic moiré is now reduced to the problem of characterizing the spatial fluctuations in the continuous-space, 2-D surface $\mathcal{D}(x)$. How we measure spatial fluctuations in this new monochrome image is arbitrary, but it is important to take into account our understanding of the human visual system to put a higher cost on mid-frequency fluctuations as opposed to very low or very high. Perhaps the easiest way to measure the visibility of stochastic moiré is to measure the visual cost³ of the stochastic moiré surface as:

$$VC(\phi_A, \phi_B) = \mathbf{E}\{AC \operatorname{Power}(\mathcal{D}(f) \times HVS(f))\},$$
(4)

where HVS(f) is the spectral, low-pass filter model of the human visual system used by Sullivan *et al.*³



Figure 3. Diagram of the spectral domain representations (a) the Poisson point process defining the sampling the circularbandlimited stochastic moiré surface with less than half the principle frequency, and (c) the power of the sampled surface with no aliasing.



Figure 4. Two examples of stochastic moiré and the corresponding moiré surfaces produced by superimposing two FM halftone patterns.

For an illustration of eqn. (4) as our new stochastic moiré metric, Fig. 4 shows two examples of stochastic moiré and the surfaces that corresponding to the halftone patterns. These particular surfaces have visual cost measures of (top) 0.0346 and (bottom) 0.0205. Here, we are assuming a 300 dpi printer and a viewing distance of 20 inches. Comparison of the two visual costs suggests an improvement in color rendition from the (top) pattern of Fig. 4 to the (bottom). Visual inspection certainly validates these results.

As a demonstration of eqn. (4)'s feasibility at quantitatively measuring stochastic moiré, we can compare the resulting visibility measures for 2-color halftoning using Floyd's and Steinberg's,⁴ Jarvis et al's,⁵ Stucki's,⁶ and Ulichney's⁷ error-diffusion. It is well documented that Floyd's and Steinberg's error-diffusion creates strong periodic textures at gray levels 1/4, 1/3, and 1/2 while Jarvis et al's does so at gray level 1/3 and Stucki at 1/3. Through a perturbation of error filter coefficients, Ulichney's errordiffusion breaks up periodic textures in the dither pattern creating a far more pleasant blue-noise halftone. In all cases, strongly correlated textures result in distinct features in the visual cost plots of Fig. 6. In Fig. 5, we show several of the corresponding stochastic moiré surfaces that are indicated in Fig. 6 by the dashed grid lines and correspond to these particularly interesting cases of moiré.

In the case of Floyd's and Steinberg's for gray levels $(g_a, g_b) = (0.25, 0.25)$ and (0.50, 0.50), the strong periodic textures of the monochrome dither patterns create large patches of fixed phase between **A** and **B** (the component dither patterns) – leading to large near-DC components in the stochastic moiré surface and, therefore, large visibility measures. A similar phenomenon occurs at gray level $(g_a, g_b) = (0.33, 0.33)$ but to a much lesser degree. Jarvis *et al*'s suffers similar correlation, to Floyd's and Steinberg's, at

gray level $(g_a, g_b) = (0.33, 0.33)$ but eliminates the high visual cost everywhere else. Stucki's follows the behavior of Jarvis *et al*'s closely but with added correlation at $(g_a, g_b) = (0.50, 0.50)$ where the visual cost is very high.



Figure 5. Selected stochastic moiré surfaces for several errordiffusion techniques.

Looking at Fig. 6, we would argue that Ulichney's error-diffusion is the best of the four for minimizing stochastic moiré visibility. Our conclusion is not based on an average or median value taken from Fig. 6 but is, instead, based on Ulichney's error-diffusion not having any extreme outliers or spikes in Fig. 6 like Floyd's and Steinberg's. Nor does Ulichney's have any wildly varying or abrupt changes in the visual cost as do the other three. What this means for the color halftones created by Ulichney's error-diffusion is that smooth gradients will show a more consistent behavior in texture.



Figure 6. Plots of the visual cost of stochastic moiré versus minority pixel intensity level for several error-diffusion schemes.

We, clearly, see this in Fig. 7 for two color gradients where a binary halftone is produced by a combining of the component halftones using a pixelwise logical OR function (1=black) where the large visibility measure of Floyd's and Steinberg's along the lines $g_a = 0.25$ and $g_b = 0.25$ is clearly apparent by observation in Fig. 7. Jarvis *et al*'s and Stucki's have the exact same problem along the $g_a = 0.33$ line and the $g_b = 0.33$ line. While the visibility measure may be lower along these lines, the halftone textures are very uncharacteristic of the rest of the halftone. What we see in Ulichney's is a halftone with no dramatic shifts texture with the same visible impression of noise across the entire pattern. What you may notice, if anything, is greater variation in texture along the diagonal where $g_a = g_b$. Surprisingly, the increased visibility of stochastic moiré along the cyan equal to magenta line is a natural property of blue-noise, and this can be seen if we note that blue-noise halftoning produces a Poisson point process and that maximizing spatial fluctuations in texture is accomplished by breaking up clusters of in-phase or clusters of out of phase pixels in the halftone. We see this in 1-D given that ϕ_A = { $a_m : m = 1, 2, ...$ } and $\phi_B = \{b_n; n = 1, 2, ...\}$ represent two Poisson point processes with an average separation between consecutive points of λ_A and λ_B , respectively. Now suppose that $a_i \in \lambda_A$ and $b_j \in \lambda_B$ are such that $a_i = b_j$, indicating that ϕ_A and ϕ_B are in-phase at a_i .



(a) Floyd and Steinberg

(b) Jarvis et al



(c) Stucki

(d) Ulichney

Figure 7. The binary halftones created by combining the two channels using a pixelwise logical OR function (1=black) for halftones created by error-diffusion.

In order for us to minimize the fluctuation in texture between ϕ_A and ϕ_B around a_i , we need to maximize the probability that a_{i+1} overlaps b_{i+1} such that the texture remains constant in the region $[a_i, a_{i+1}]$ where, for uncorrelated ϕ_A and ϕ_B , $\mathbf{Pr}(a_{i+1} = b_{i+1})$ is equal to:

$$\int_{x=a_{\cdot}}^{+\infty} \mathbf{Pr}(a_{i+1}=x) \cdot \mathbf{Pr}(b_{j+1}=x) dx.$$
 (5)

From eqn. (5), the probability that a_{i+1} overlaps b_{j+1} is maximized when $\lambda_A = \lambda_B$ for uncorrelated ϕ_A and ϕ_B . It is, therefore, stated that since λ_A and λ_B are functions of their corresponding minority pixel intensities, fluctuations in texture are minimized when the intensities of two component dither patterns are equal. Since we are weighting low frequency artifacts, not DC, with greater weight than high, the corresponding visual cost of stochastic moiré will be maximized.

3. Conclusions

Stochastic moiré refers to the low-frequency graininess exhibited by overlapping FM halftone patterns. In a previous paper, we showed that measuring or characterizing stochastic moiré can be accomplished by measuring the shortest distance between the minority pixels of the component patterns. From this, we derived a continuous-space surface that completely characterizes the fluctuations in texture that occur between the overlapping patterns. In this paper, we showed that the visibility of stochastic moiré is its most visible when the component patterns have the same spacing between dots, and using several traditional errordiffusion schemes, we calculated the moiré visibility of two color halftones and could see noticeable artifacts in the halftone that directly correlate with high visibility measures.

Given the natural tendency of blue-noise to maximize stochastic moiré visibility when two superimposed dither patterns have the same intensity, minimizing stochastic moiré visibility in these regions requires either that the intensity of one pattern be manipulated or that the statistical character of one of the patterns be varied. The second, and preferred approach, can be achieved by adding noise to the image prior to halftoning in these regions of equal intensity such that the average intensity, and hence the average hue, does not change.⁸ This can also be achieved by perturbing the quantization threshold used in error diffusion with low variance white-noise in these same regions. The overall effect is to "whiten" the spectral content of the corresponding pattern,⁹ and thereby, increase the variability in the pattern with a minority pixel less likely to be found an exact distance _A from its nearest neighbors.

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4. Biography

Dr. Daniel L. Lau received his B.S. in Electrical Engineering from Purdue University, West Lafayette with highest distinction in 1995 and then his Ph. D. degree in Electrical Engineering from the University of Delaware in 1999. His published works in halftoning include an article in the December 1998 issue of the *Proceedings of the IEEE* and his own book *Modern Digital Halftoning* published by Marcel Dekker, January 2001. Daniel is a regular contributer to the IS&T's PICS conference, and is, currently an Assistant Professor at the University of Kentucky where he is continuing his work in digital halftoning and color reproduction. Other areas of image processing that are of interest to Daniel include digital watermarking, object recognition, and 3D video.